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This blog shows a derivation of a time domain model of a BLDC PMSM motor. The block diagram is shown in Figure 1.

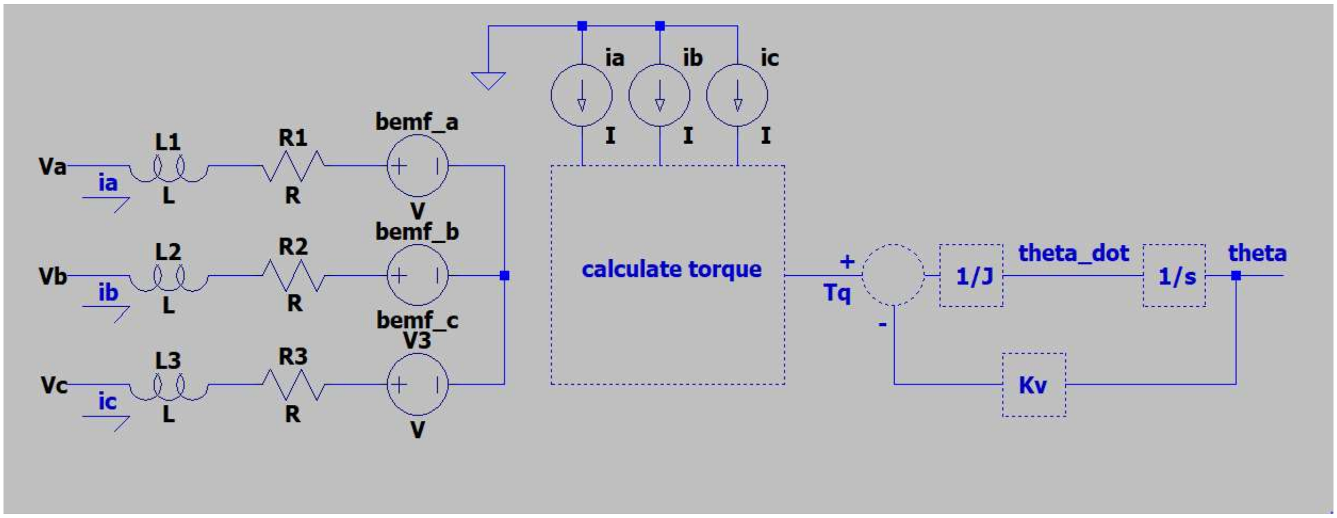

Figure 1 – BLDC PMSM model

Derivation

The model assumes a 3-phase Np pole motor is driven with a sinusoidal drive which can an averaged model of Space Vector Modulation (SVM) to reduce simulation time, or a switched SVM model.

The equations can be divided into two sets, the first is the electrical input and the second is the mechanical generation of torque. The electrical equations are given here by using Kirchoff's Law and summing voltages around a closed loop and net current into a node. In these equations all currents and voltages are functions of time, they do not yet assume what the input voltage waveform is.

$$Va - L \cdot \frac{dia}{dt} - i_a \cdot R - bemf_a + bemf_b + i_b \cdot R + L\frac{di_b}{dt} = Vb$$
$$Vc - L \cdot \frac{di_c}{dt} - i_c \cdot R - bemf_c + bemf_a + i_a \cdot R + L\frac{di_a}{dt} = Va \quad [1]$$
$$i_a + i_b + i_c = 0$$

The inputs for the electrical portion are the bus voltages, Va(t), Vb(t) and Vc(t). The outputs are the phase currents i_a*(t)*, i_b(t), and i_c(t). The bemf voltages are coupled from the mechanical model.

The mechanical model is given by equations [2],

$$\dot{\omega}(t) = torque(t) - \frac{Kv}{J}\omega(t) \quad [2]$$

Equations [1] and [2] are rewritten below in a form to explicitly show the state variables we will solve. The states of the system are ia(t), ib(t), ic(t), θ(t) and ω(t).

$$\begin{aligned}
\frac{di_a}{dt} &= \frac{1}{3L} \cdot (2V_a - V_b - V_c - 3i_aR - 2 \cdot bemf_a + bemf_b + bemf_c) \\
\frac{di_b}{dt} &= \frac{1}{3L} \cdot (-V_a + 2V_b - V_c - 3i_bR + bemf_a - 2bemf_b + bemf_c) \\
\dot{\omega} &= Torque(t) - \frac{Kv}{J}\omega \\
\dot{\theta} &= \omega \\
0 &= i_a + i_b + i_c
\end{aligned} \qquad [3]$$

We will talk about the torque and bemf terms next. Both of these are also dependent on the rotor flux.

Torque and Bemf

When the motor winding's are not connected and the motor is spun externally, the rotor magnets will induce a sinusoidal voltage in the three stator winding's (armature) which we denoted above as $bemf_a$, $bemf_b$ and $bemf_c$. These three voltage are displaced from one another by 120 electrical degrees. The rotor flux is a vector $\boldsymbol{\psi}_{\mathbf{rotor}}$ rotating at the rotor speed.

When the stator coils A, B & C are energized there will be a net flux $\boldsymbol{\psi}_{arm}$ generated by the coils that attracts the opposite polarity rotor flux. When the coils are driven with sinusoidal voltages displaced 120 degrees apart then $\boldsymbol{\psi}_{arm}$ will be a vector that rotates at the frequency determined by the controller. This will move the motor until the rotor flux and armature flux are aligned and the torque is at a null. The torque that spins the motor is maximum when the armature flux is pointing 90 electrical degrees from the rotor flux. In general the bemf is given by Faraday's law as

$$\varepsilon = -n\frac{d\phi}{dt} \qquad [4]$$

When the angle between the rotor and stator is λ, the shaft torque generated by the motor will be

$$Torque_q = (\Psi_{rotor})(\Psi stator)sin(\lambda) \qquad [5]$$

The q subscript on Torque is to denote torque along the quadrature axis. This is the shaft torque that generates work.

When λ is 90 electrical deg the generated torque will be maximized. The bemf generated by a sinusoidal rotor flux is given by the following,

$$\begin{aligned}
bemf_a(t) &= K_b \cdot \omega \cdot sin(\omega t - \lambda) \\
bemf_b(t) &= K_b \cdot \omega \cdot sin(\omega t - \frac{2}{3}\pi - \lambda) \\
bemf_c(t) &= K_b \cdot \omega \cdot sin(\omega t - \frac{4}{3}\pi - \lambda)
\end{aligned} \qquad [6]$$

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The conservation of power across the air gap holds so the product of winding current and bemf voltage equals the product of the rotor speed and torque generated by that winding. As a consequence the winding currents will also be sinusoidal.

$$\begin{aligned} i_a(t) &\propto sin(\omega t - \lambda) \\ i_b(t) &\propto sin(\omega t - \frac{2}{3}\pi - \lambda) \\ i_c(t) &\propto sin(\omega t - \frac{4}{3}\pi - \lambda) \end{aligned} \qquad [7]$$

SVM & Field Oriented Control

In the next section (or follow-on blog) we will develop the foundation of FOC control and show the spatial transformations needed to control the stator flux and rotor flux amplitudes independently and control the angle λ between them.