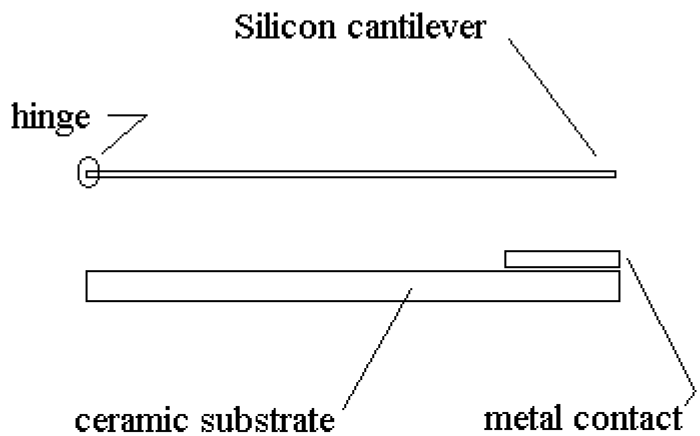
The problem being presented here is:

- Formulating the dynamic equations of a hypothetical MEMS device,
- Deriving a mapping equation which allows global linearization of the dynamics, and
- Showing a closed loop control solution to the MEMS positioner.

Hypothetical MEMS device,



Hinge stiffness	ks
Inertia	J
Viscous damping	kb
Spacing	h
Cantilever length	L
Cantilever width	W
Contact length	We
Contact width	W

Assumptions

- h/L < 15%
- Rigid body motion.
- Single degree of freedom, rotation around hinge in angle θ.
- Cantilever at ground potential.
- Contact at potential v volts.
- Voltage driver, (not charge controlled).

Calculate Capacitance

- From first principles $dC = \varepsilon \cdot \frac{dA}{dh}$

- $dC = \varepsilon \cdot \frac{W \cdot d\lambda}{h - L \cdot \theta + \lambda \cdot \theta}$, $\lambda = 0..\mathrm{W_e}$

- $C = \frac{\varepsilon \cdot W}{\theta} \cdot \log\left(\frac{h\text{-}(L\text{-}W_e) \cdot \theta}{h - L \cdot \theta}\right)$

Calculate Potential Energy

- From first principles $U = \frac{1}{2} \cdot C \cdot v^2$

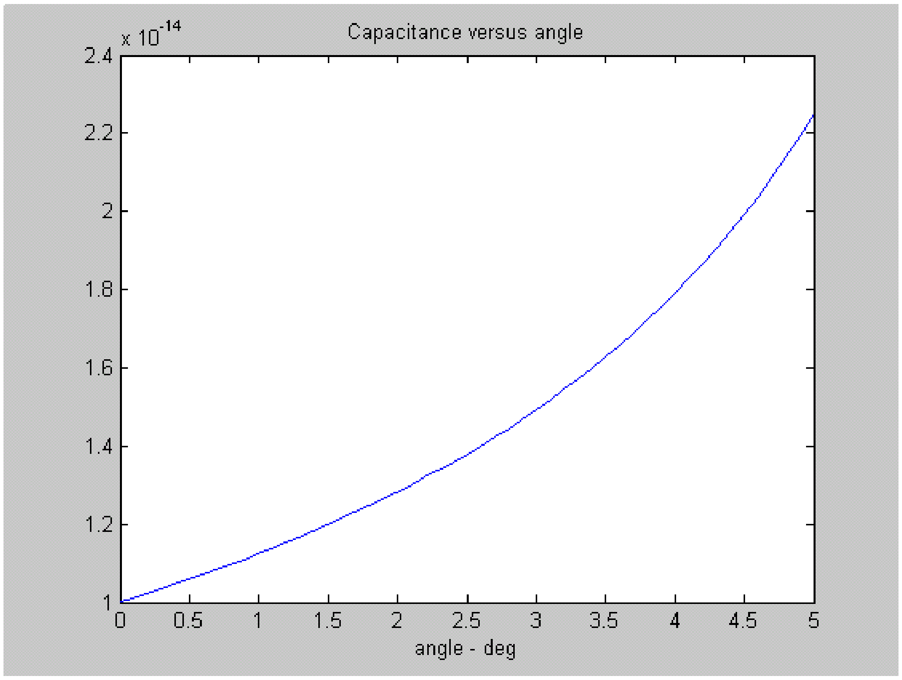
- $U = \frac{\varepsilon \cdot W \cdot v^2}{2 \cdot \theta} \cdot \log\left(\frac{h\text{-}(L\text{-}W_e) \cdot \theta}{h - L \cdot \theta}\right)$

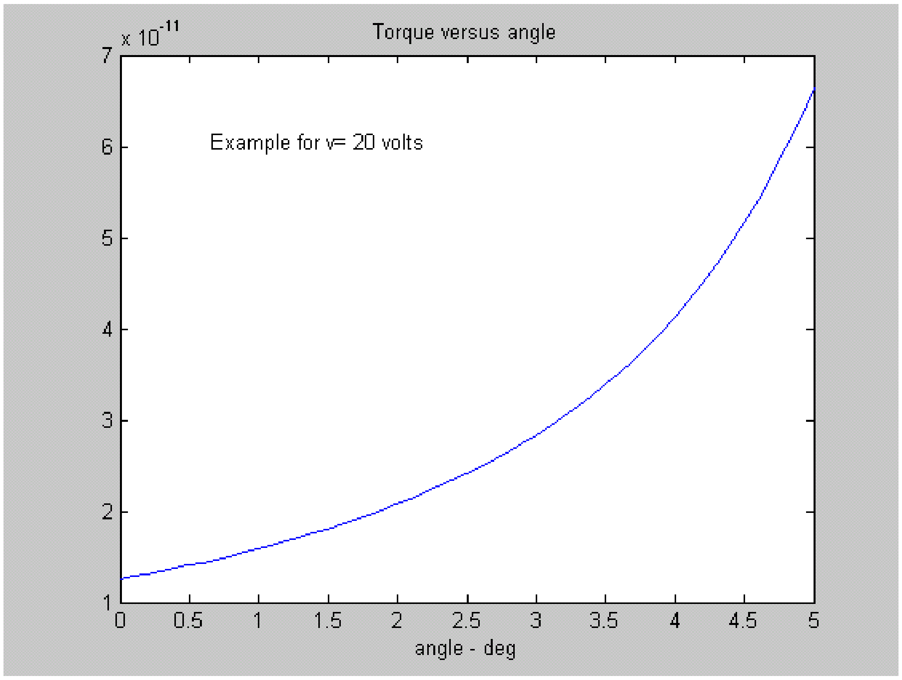
Calculate Electrostatic Torque

- From first principles $\Gamma = \frac{dU}{d\theta}$

- $\Gamma = \frac{\varepsilon \cdot W \cdot v^2}{2 \cdot \theta} \cdot \left[\frac{h \cdot W_e}{\left(h - (L - W_e) \cdot \theta\right) \cdot \left(h - L \cdot \theta\right)} - \frac{\log\left(\frac{h\text{-}(L\text{-}W_e) \cdot \theta}{h - L \cdot \theta}\right)}{\theta}\right]$

Examples


Capacitance versus angle
x 10^-14
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
angle - deg


Torque versus angle
x 10^-11
Example for v= 20 volts
7
6
5
4
3
2
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
angle - deg

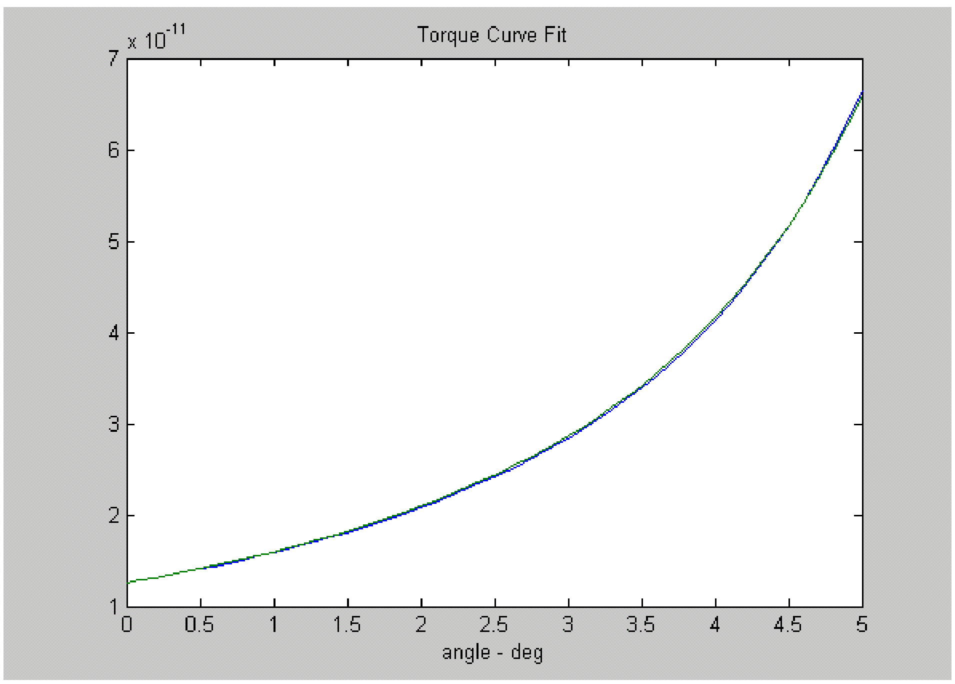
Approximation to Torque Equation

The torque equation as derived, repeated here, is nonlinear and not very tractable for real-time control.

$$\Gamma = \frac{\varepsilon \cdot W \cdot \nu^2}{2 \cdot \theta} \cdot \left[\frac{h \cdot W_e}{\left(h - (L - W_e) \cdot \theta\right) \cdot \left(h - L \cdot \theta\right)} - \frac{\log\left(\dfrac{h\text{-}(L\text{-}W_e) \cdot \theta}{h - L \cdot \theta} \right)}{\theta} \right]$$

An approximation to the torque equation is found by curve fitting to be,

$$\Gamma = \frac{k_1 \cdot \nu^2}{\left(1 - k_2 \cdot \theta\right)^2}$$



<u>Dynamic equations of motion</u>:

$$\ddot{\theta} = -\frac{ks}{J} \cdot \theta - \frac{kb}{J} \cdot \dot{\theta} + \frac{T(\nu,\theta)}{J}$$

The small signal stiffness is given by,

$$Kss = ks - \frac{dT(\nu,\theta)}{d\theta}$$

- At small angles the stiffness is just the hinge stiffness Kss= ks.
- At mid-range angles the stiffness will be approximately Kss=0. This defines the flip-over point.
- Beyond flip-over the stiffness is increasingly negative.

A control system that doesn't take into account the above nonlinear behavior will have a transient response that depends on the operating point and will be sub-optimal.

Global Linearization

Define a pseudo control signal as follows,

$$\xi \equiv T(\nu, \theta)$$

Now the system dynamics in ξ input-space can be defined as,

$$\ddot{\theta} = -\frac{ks}{J} \cdot \theta - \frac{kb}{J} \cdot \dot{\theta} + \frac{\xi}{J}$$

which is a linear system.

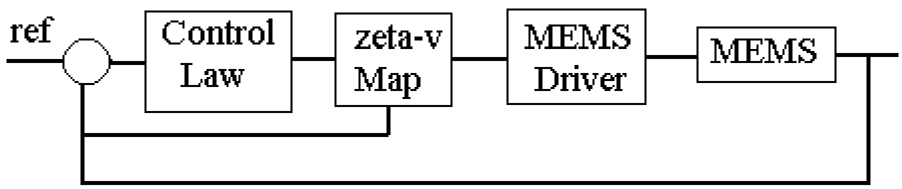
Calculation of v given ξ

The linearization is a mapping from pseudo control ξ to voltage v given by

$$\nu \equiv \left(\frac{\xi}{k_1} \right)^{1/2} \cdot (1 - k_2 \cdot \theta)$$

This mapping is easily implemented in DSP firmware and calculated every sample time.

Control System Block Diagram



- Measure angle at each sample and calculate position error.
- If using Kalman filter update position and velocity estimate.
- Control Law calculates pseudo control Zeta from position, velocity and integral of position error.
- Calculate mapping from Zeta to v, using current angle.
- Output v to DAC.
- Calculate position and velocity prediction of using Kalman filter.
- Return from interrupt routine.